Saturated Hydraulic Conductivity expressed as $pK_s$

Bernhard Buchter, Peter Richard Zünd
Alberta GmbH, Bertastrasse 18a, CH-8003 Zürich, Switzerland

**Abstract**
This technical article proposes that Saturated Hydraulic Conductivity be expressed as a log to the base 10, ($pK_s = -\log_{10}(K_s[m/s])$) and that specific values for $pK_s$ be used depending on the error of measurement.

Keywords: Saturated Hydraulic Conductivity, $pK_s$, standard values.

**Introduction**
Hydraulic conductivity is a key factor in soil science together with the water characteristic curve. Whereas the values of the latter are quite easily comprehensible, values of hydraulic conductivity are not. Conductivity in soils has a very large range, usually between $10^{-3}$ and $10^{-7}$ m/s in most soils, and in packed gravel $10^{-2}$ m/s and in landfill liners $10^{-9}$ m/s.

Measurement is difficult due to the very large range in conductivity and also due to the sensitivity of the measurement equipment. For example, a soil sample with a diameter of 5.5 cm and a conductivity of $10^{-3}$ m/s has a water flow rate of 2.5 ml/s (gradient 1). A sample with the same size and a conductivity of $10^{-4}$ m/s has a water flow rate of 1.5 ml/h, whereas a sample with a conductivity of $10^{-7}$ m/s has a water flow rate of 0.9 ml/h. For the most permeable example ($K_s = 10^{-3}$ m/s), the flow rate is close to the conductivity of the measuring apparatus, due to restrictions caused by the diameter of the connecting tubes, and thus close to the upper limit of the measurement range. For the least permeable example ($K_s = 10^{-7}$ m/s), the flow rate is close to the lower limit of the apparatus, due to evaporation from the input and output tanks and other possible leakage.

**Discussion**
Values of hydraulic conductivity should be reported using SI-units, (i.e. m/s) as pointed out by Baer (1987), however commonly a variety of different units are used by scientists, like m/day (Szilagyi, 2004; Baer, 1987), cm/day (Lebert and Böken, 2008; Mecke et al., 2000; Ruiz and Medina, 2003), cm/s (Baer, 1987), mm/h (Bagarello et al., 2000; van Dijk, 2002), mm/s (Esteves et al. 2005), and sometimes inch/h and feet/day (Van Dijk, 2002 and Iowa Stormwater Partnership). The reason for the variety of units being used is due to the fact that hydraulic conductivity expressed as m/s is difficult to comprehend.
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In groundwater studies often only the exponent of the value is used to characterize the hydraulic conductivity (Baer, 1987). But despite Baer’s (1987) comment that SI units (m/s) should be used, Baer (1987) expresses the negative exponent of the hydraulic conductivity in cm/s. Problems with very small values arise not only in soil science, but also in chemistry. Concentrations of H₃O⁺ ions are as low as 10⁻¹⁴ Mol/L in an alkaline environment and go up to 10⁻¹ Mol/L in an acidic environment, i.e. pH 1 to 14.

Therefore it would be helpful to use the same notation for Saturated Hydraulic Conductivity (Kₛ),

\[ pK_s = -\log_{10}(K_s[m/s]). \]

With this notation it is mandatory to use the unit m/s. By taking the logarithm to the base 10 and multiplying by -1, the values of pKₛ in soils are then commonly between 3 and 7. This notation is straightforward and by far easier to comprehend than any other non-logarithmized notation. The arithmetic mean of pKₛ values, i.e. the mean of the logarithmized values, can easily be calculated and is more appropriate than the arithmetic mean of Kₛ itself, because Saturated Hydraulic Conductivity is log-normally distributed (Nielsen et al., 1973).

The pKₛ notation also prevents us from creating too many narrow classes, which are hardly justifiable given the problems of error encountered when measuring Saturated Hydraulic Conductivity. The most reasonable classification is the division of pKₛ into steps of 1, corresponding to steps of 1 order of magnitude for Kₛ (Klute and Dirksen, 1986).

Depending on the estimated or calculated error (precision) of measurement, a limited range of measured values should be used (Table 1.). The standardized values with their tolerance range span the full range of conductivities possible. Standardised values depending on tolerance or precision are commonly use for resistors in electronics. The larger the error, the greater are the intervals between the standardised values.

The number of intervals per order of magnitude for Kₛ can be calculated as follows:

\[ n = 1/\log((1+f)/(1-f)) = 1/(\log((1+f)^2)) \]

Where \( f = \Delta x/x \) is the relative error and \( \Delta x \) the absolute error of the measured value \( x \).
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Table 1 Standardized values of $pK_s = -\log_{10}(K_s[m/s])$ and corresponding values of $K_s$ depending on the relative error [%] of $K_s$, where $x = \ldots, 2, 3, 4, 5, 6, 7, \ldots$ and $y = x + 1$.

$pK_s$: Saturated Hydraulic Conductivity expressed as a log to the base 10.

$K_s$: Saturated Hydraulic Conductivity.

<table>
<thead>
<tr>
<th>pKs</th>
<th>Relative error [%]</th>
<th>100</th>
<th>50</th>
<th>20</th>
<th>10</th>
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<tr>
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</tr>
<tr>
<td>x.0</td>
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<tr>
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</table>

For measurement errors of 100 %, 50 %, 20 % and 10 %, equal to relative errors $f = 1, 0.5, 0.2$ and 0.1, the number of specified values per order of magnitude are $n = 2, 3, 6$ and 12. If $n = 6$ and 12 are replaced with $n = 5$ and 10, the resulting standardized values for $pK_s$ become simpler. The standardized values are then obtained from

$$pK_s(i+1) = pK_s(i) + 1/n.$$  

For example, with a measurement error of 20 %, the preferred values of $pK_s$ increase in intervals of 0.2, e. g. 3.2, 3.4, 3.6 and 3.8, or in general, x.2, x.4, x.6 and x.8 with $x = \ldots, 2, 3, 4, 5, 6, 7, \ldots$ These $pK_s$ values correspond to $K_s = 6.3, 4.0, 2.5$ and $1.6 \times 10^3$ m/s where $y = x + 1$. With an error of 100, 50, and 10, there are 2, 3 and 10 standardized values for each order of magnitude of $pK_s$. Table 1 summarizes the preferred values.

Finally, the number of significant figures depends on the measurement error as proposed in Table 1.

By using $pK_s = -\log_{10}(K_s[m/s])$ instead of $K_s$, the problem of units will disappear and thus simplify the discussion of Hydraulic Conductivity among soil specialists giving more time to focus on solving the many problems encountered when measuring hydraulic conductivity.
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References